



Calcul intégral

2^{ème} Bac Pro

Correction des exercices du polycopié du cours

EX.1 Vérifier les calculs suivants:

1°/ $\int_{-1}^0 (2x+1) dx = 0$

2°/ $\int_0^1 (x^3 + 6x + 1) dx = \frac{13}{4}$

3°/ $\int_4^{16} \frac{dx}{2\sqrt{x}} = 2$; 4°/ $\int_0^3 \frac{dx}{x+2} = \ln\left(\frac{5}{2}\right)$

Solution: 1°/ $\int_{-1}^0 (2x+1) dx = ? = 0$

on a: $\int_{-1}^0 (2x+1) dx = \int_{-1}^0 f(x) dx$

avec: $f(x) = 2x+1$

une primitive de f : $F(x) = x^2 + x$

car: $F'(x) = (x^2 + x)' = 2x + 1$

donc: $\int_{-1}^0 (2x+1) dx = \int_{-1}^0 F'(x) dx$

$= [F(x)]_{-1}^0 = [x^2 + x]_{-1}^0$

$= 0^2 + 0 - ((-1)^2 + (-1)) = -(1-1) = 0$

2°/ $\int_0^1 \underbrace{(x^3 + 6x + 1)}_{f(x)} dx = ? = \frac{13}{4}$

$\Rightarrow F(x) = \frac{x^3}{3} + 6\frac{x^2}{2} + x$; F est une primitive.
donc:

$\int_0^1 f(x) dx = [F(x)]_0^1 = F(1) - F(0)$

avec: $\begin{cases} F(1) = \frac{1}{3} + \frac{6}{2} + 1 = \frac{1}{3} + 4 = \frac{13}{3} \\ F(0) = 0 + 0 + 0 = 0 \end{cases}$

donc: $\int_0^1 (x^3 + 6x + 1) dx = \frac{13}{3}$

3°/ $\int_4^{16} \frac{dx}{2\sqrt{x}} = \int_4^{16} \underbrace{\frac{1}{2\sqrt{x}}}_{f(x)} dx = ? = 2$

①

$f(x) = \frac{1}{2\sqrt{x}} \Rightarrow F(x) = \sqrt{x}$

donc: $\int_4^{16} f(x) dx = [\sqrt{x}]_4^{16} = \sqrt{16} - \sqrt{4} = 4 - 2 = 2$

4°/ $\int_0^3 \frac{dx}{x+2} = \int_0^3 \frac{1}{x+2} dx = ? = \ln\left(\frac{5}{2}\right)$

$f(x) = \frac{1}{x+2} = \frac{(x+2)'}{x+2} = \ln'(x+2)$

$F(x) = \ln(x+2)$ donc:

$\int_0^3 \frac{dx}{x+2} = F(x) \Big|_0^3 = [\ln(x+2)]_0^3$

$= \ln(3+2) - \ln(0+2) = \ln 5 - \ln 2 = \ln\left(\frac{5}{2}\right)$

EX.2 Montrer que:

1°/ $\int_0^\pi (4x + \frac{2}{3} \sin(x)) dx = 2\pi^2 + \frac{4}{3}$

2°/ $\int_0^1 (5x^3 + e^x) dx = e + \frac{1}{4}$

3°/ $\int_1^e (\frac{1}{x} - \frac{1}{x^2}) dx = \frac{1}{e}$

4°/ $\int_1^2 (2x+3)(x^2+3x)^2 dx = 312$

Solution:

1°/ $\int_0^\pi (4x + \frac{2}{3} \sin(x)) dx$
 $= 4x \int_0^\pi x dx + \frac{2}{3} \int_0^\pi \sin(x) dx$

$= 4 \left[\frac{x^2}{2} \right]_0^\pi + \frac{2}{3} [-\cos(x)]_0^\pi$

$= 4 \left[\frac{\pi^2}{2} - 0 \right] + \frac{2}{3} [-(-1) + 1]$

$= 4 \times \frac{\pi^2}{2} + \frac{2}{3} \times 2 = 2\pi^2 + \frac{4}{3}$

2°/ $\int_0^1 (5x^3 + e^x) dx = 5 \int_0^1 x^3 dx + \int_0^1 e^x dx$
 $= 5 \left[\frac{x^4}{4} \right]_0^1 + [e^x]_0^1 = 5 \left(\frac{1}{4} - 0 \right) + e^1 - e^0$

$$\int_0^1 (5x^3 + e^x) dx = \frac{5}{4} + e - 1$$

$$= e + \frac{5}{4} - 1 = e + \frac{1}{4}$$

$$30/ \int_1^e \frac{1}{x} - \frac{1}{x^2} dx = \int_1^e \frac{1}{x} dx + \int_1^e \frac{1}{x^2} dx$$

$$= [\ln(x)]_1^e + \left[\frac{1}{x}\right]_1^e$$

$$= \ln(e) - \ln(1) + \frac{1}{e} - \frac{1}{1}$$

$$= 1 - 0 + \frac{1}{e} - 1 = \boxed{\frac{1}{e}}$$

car: $\ln'(x) = \frac{1}{x}$ et $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

donc: $\frac{1}{x} - \frac{1}{x^2} = \left(\ln(x) + \frac{1}{x}\right)'$

$$40/ \int_1^2 \underbrace{(2x+3)}_{f(x)} (x^2+3x)^2 dx$$

$$f(x) = (x^2+3x)'(x^2+3x)^2$$

$$= \frac{1}{3} \times [3(x^2+3x)'(x^2+3x)^2]$$

$$= \frac{1}{3} F'(x); \text{ avec: } F(x) = (x^2+3x)^3$$

donc: $\int_1^2 f(x) dx = \frac{1}{3} [F(x)]_1^2 = \frac{1}{3} (F(2) - F(1))$

$$\begin{cases} F(2) = (2^2 + 3 \times 2)^3 = (4+6)^3 = 1000 \\ F(1) = (1+3)^3 = 4^3 = 64 \end{cases}$$

$$\int_1^2 f(x) dx = \frac{1}{3} [1000 - 64] = \frac{936}{3} = \boxed{312}$$

EX. 3 Montrer que:

$$10/ \int_0^3 2x e^{x^2} dx = e^9 - 1$$

$$20/ \int_1^2 \sqrt{x} dx = \frac{4\sqrt{2}}{3} - \frac{2}{3}$$

$$30/ \int_1^2 \frac{2x+3}{x^2+3x} dx = \ln\left(\frac{5}{2}\right)$$

Solution: 10/ $\int_0^3 2x e^{x^2} dx$

$$f(x) = (x^2)' e^{x^2} = (e^{x^2})' = F'(x)$$

avec: $F(x) = e^{x^2}$ donc:

$$\int_0^3 f(x) dx = [F(x)]_0^3 = F(3) - F(0)$$

$$= e^9 - e^0 = e^9 - 1 = \boxed{e^9 - 1}$$

$$20/ \int_1^2 \sqrt{x} dx = \int_1^2 x^{1/2} dx$$

$$= \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_1^2 = \left[\frac{x^{3/2}}{3/2} \right]_1^2$$

$$= \frac{2^{3/2+1}}{3/2} - \frac{1}{3/2} = \frac{2^{5/2} \times 2}{3/2} - \frac{2}{3}$$

$$= \sqrt{2} \times 2 \times \frac{2}{3} - \frac{2}{3} = \boxed{\frac{4}{3}\sqrt{2} - \frac{2}{3}}$$

$$30/ \int_1^2 \frac{2x+3}{x^2+3x} dx = \int_1^2 \frac{(x^2+3x)'}{x^2+3x} dx$$

$$= \int_1^2 \ln'(x^2+3x) dx = [\ln(x^2+3x)]_1^2$$

$$= \ln(2^2+3 \times 2) - \ln(1+3 \times 1)$$

$$= \ln(10) - \ln(4) = \ln\left(\frac{10}{4}\right) = \boxed{\ln\left(\frac{5}{2}\right)}$$

EX. 4 A l'aide d'une intégrale

par partie montrer que:

$$10/ \int_{-1}^2 x e^x dx = 0$$

$$20/ \int_0^\pi x \sin(x) dx = \pi$$

$$30/ \int_2^e 4x^3 \ln(x) dx = 3\frac{e^4}{4} - 16\ln(2) + 4$$

Solution:

$$10/ \int_{-1}^2 x e^x dx = \int_{-1}^2 u(x) v'(x) dx$$

avec: $\begin{cases} u(x) = x \\ v'(x) = e^x \end{cases}$ donc: $\begin{cases} u'(x) = 1 \\ v(x) = e^x \end{cases}$

par suite: $\int u(x)v'(x)dx = [u(x)v(x)] - \int u'(x)v(x)dx$

$$\begin{aligned}\Rightarrow \int_{-1}^2 x e^x dx &= [1x e^x]_{-1}^2 - \int_{-1}^2 1x e^x dx \\ &= e^2 - e^{-1} - \int_{-1}^2 e^x dx \\ &= e^2 - e^{-1} - [e^x]_{-1}^2 = e^2 - e^{-1} - (e^2 - e^{-1}) \\ &= e^2 - e^{-1} - e^2 + e^{-1} = \boxed{0}\end{aligned}$$

2°/ $\int_0^\pi x \sin(x) dx = \int_0^\pi \underbrace{x}_{u(x)} \underbrace{\sin(x)}_{v'(x)} dx$

$$\begin{cases} u(x) = x \\ v'(x) = \sin(x) \end{cases} \Rightarrow \begin{cases} u'(x) = 1 \\ v(x) = -\cos(x) \end{cases}$$

donc

$$\begin{aligned}\int u(x)v'(x)dx &= [u(x)v(x)] - \int u'(x)v(x)dx \\ \int_0^\pi x \sin(x) dx &= [-x \cos(x)]_0^\pi - \int_0^\pi 1x (-\cos(x)) dx \\ &= [-\pi(-1) + 0] + \int_0^\pi \cos(x) dx \\ &= \pi + [\sin(x)]_0^\pi = \pi + (0 - 0) = \boxed{\pi}\end{aligned}$$

3°/ $\int_2^e 4x^3 \ln(x) dx = \int_2^e (4x^3) \ln(x) dx$

$$\begin{cases} u'(x) = 4x^3 \\ v(x) = \ln(x) \end{cases} \Rightarrow \begin{cases} u(x) = x^4 \\ v'(x) = \frac{1}{x} \end{cases}$$

$$\begin{aligned}\int_2^e 4x^3 \ln(x) dx &= [u(x)v(x)]_2^e - \int_2^e u(x)v'(x) dx \\ &= [x^4 \ln(x)]_2^e - \int_2^e x^4 \times \frac{1}{x} dx \\ &= e^4 \ln(e) - 2^4 \ln(2) - \int_2^e x^3 dx \\ &= e^4 \times 1 - 16 \ln(2) - \left[\frac{x^4}{4} \right]_2^e \quad \textcircled{3}\end{aligned}$$

$$\begin{aligned}&= e^4 - 16 \ln(2) - \left(\frac{e^4}{4} - \frac{2^4}{4} \right) \\ &= e^4 - \frac{e^4}{4} - 16 \ln(2) + \frac{16}{4} \\ &= \left(1 - \frac{1}{4} \right) e^4 - 16 \ln(2) + 4 \\ &= \boxed{\frac{3}{4} e^4 - 16 \ln(2) + 4}\end{aligned}$$